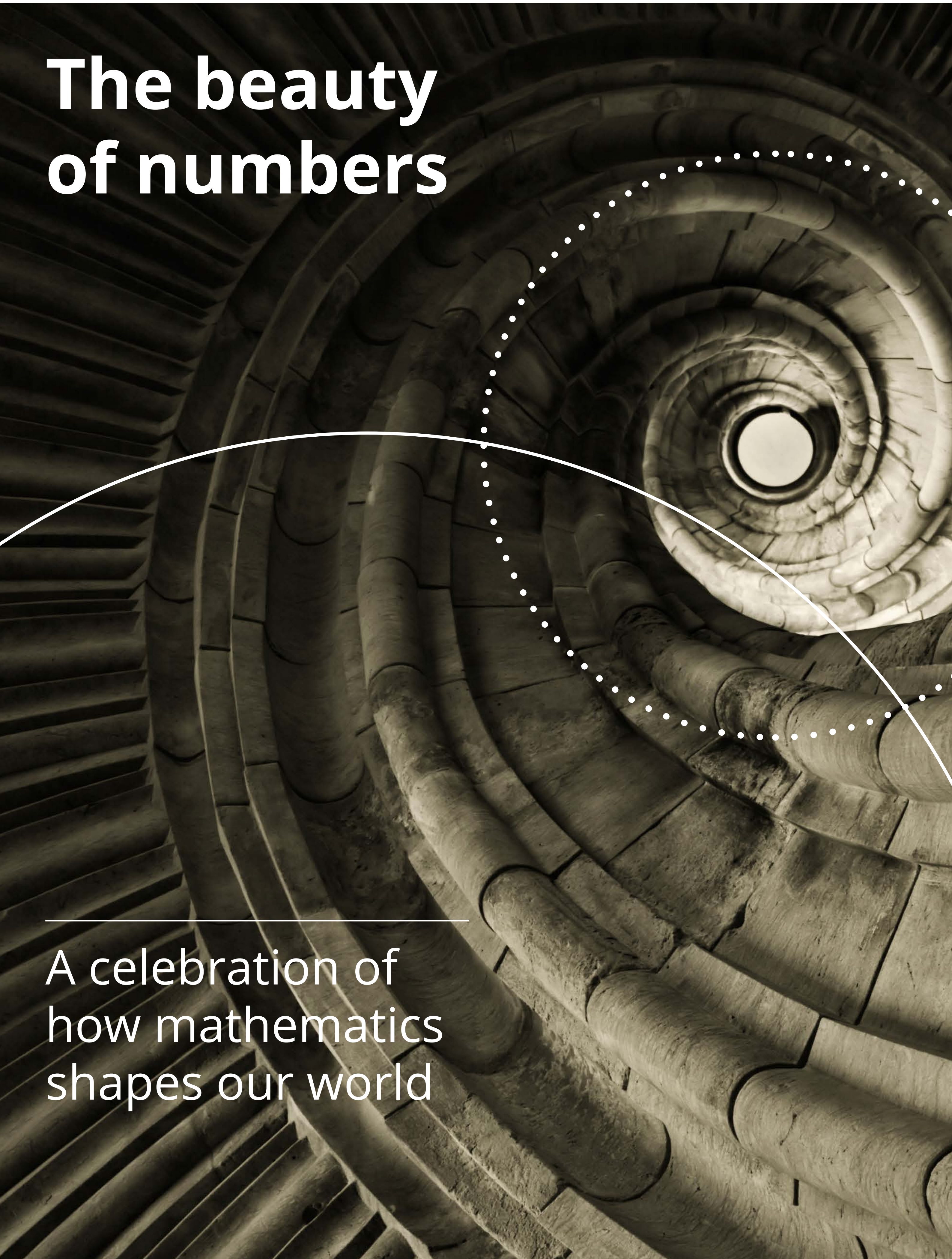
WILEY



The beauty of numbers

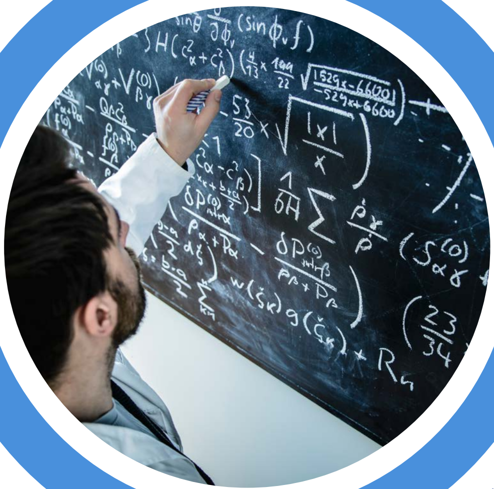
A celebration of how mathematics shapes our world

WILEY

Table of Contents

Introduction

The idea that mathematics can be beautiful will hardly come as a surprise to most practitioners. Anyone who has a natural affinity for the subject appreciates that numbers, shapes, equations and formulae have an allure extending far beyond the purely intellectual realm.



It's perhaps fitting, then, that mathematicians often describe theorems and proofs as 'elegant' when they exhibit an appealing succinctness and clarity. On the other hand, methods that are technically correct, yet lack finesse, are invariably labelled 'clumsy' or even 'ugly'[1].

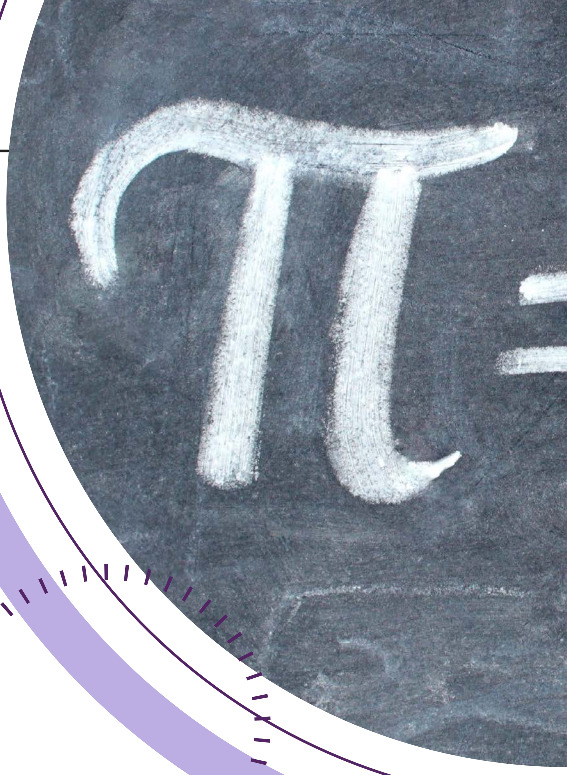
Experts across multiple fields have attempted to reconcile the apparent discord between the objectivity of mathematics and the subjectivity of appraising beauty. The results of their endeavours have not only helped us further our understanding of our planet and the universe but also produced some of the most stunning artistic masterpieces known to man.

In this guide, Francis Finch examines the ways in which mathematics can be beautiful and explores how key mathematical concepts are replicated in nature and represented across art, music and architecture.

1 Page 150, Patronizing the Arts. Marjorie Garber.
https://books.google.co.uk/books?id=e7T1ZgHxRu0C&pg=PA150&lpg=PA150&dq=clumsy+ugly+elegant+mathematics&source=bl&ots=RWhgw7TJiu&sig=Mgwo-0YD_hZR-33Ihk7LBKOGknM&hl=en&sa=X&ved=0ahUKEwjasti3qbHKAhVLgpAKHaZEBB8Q6AEIMzAE#v=onepage&q=clumsy%20ugly%20elegant%20mathematics&f=false

WILEY

What's in a number?



Pick a number, any number. This exercise is usually the precursor to magic tricks or parlour games, but it can also provide an interesting insight into our relationship with numbers. Many people who are instructed to select a number often choose their favourite or 'lucky' digit.

A recent global survey from Brazilian mathematician Alex Bellos claimed '7' is the world's favourite number[2], based on responses from more than 30,000 people.

But would you get the same result if you asked mathematicians for their favourite number? Probably not. In fact, they are likely to choose from a multitude of numbers that could be considered ugly to the untrained eye, yet hold specific mathematical significance. For example, pi (π) and the Golden Ratio (Φ) are irrational numbers, but both are frequently cited for their beauty. Other popular choices could include Kaprekar's constant (6174) the Hardy-Ramanujan number (1729) or 36543454565434563, which is currently acknowledged as the largest palindromic triangular number with a palindromic index[4].

These numbers all possess unique quirks and properties that may make them beautiful to a mathematician. Moreover, numbers are just one of many mathematical building blocks. It is perhaps easier to understand the natural beauty in mathematics once the scope is widened to include more complex mathematical concepts.

2 http://pages.bloomsbury.com/favouritenumber

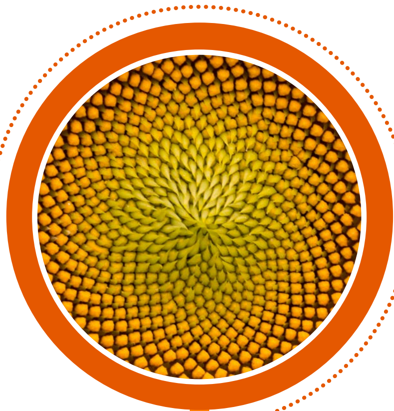
3 http://www.theguardian.com/science/alexs-adventures-in-numberland/2014/apr/08/seven-worlds-favourite-number-online-survey

4 Page 278. Figurate Numbers. Elena Deza, Michel Marie Deza

Beauty and the brain

Beauty is inherently subjective; people can differ wildly on what they believe to be aesthetically pleasing based on their culture, upbringing, education, personal tastes and more. 'Beauty is in the eye of the beholder' is a common phrase used to encapsulate this subjectivity.

However, research has indicated that beauty can be objectively identified and quantified, at least in part, through brain activity[5]. Professor Semir Zeki and Dr Tomohiro Ishizu from the Wellcome Laboratory of Neurobiology at University College London discovered in 2011 that certain pieces of art and music have a measurable impact on the section of the brain related to pleasure and reward.



The researchers asked 21 volunteers to rate excerpts of art and music as either 'beautiful', 'indifferent' or 'ugly'. Participants were then asked to view or listen to the examples again while lying in a functional magnetic resonance imaging (fMRI) scanner to measure their brain activity. The experiment revealed that the medial orbitofrontal cortex, a region at the front of the brain, lit up when people saw or heard art and music they had previously rated 'beautiful'. Brain activity also increased in the caudate nucleus region, which is commonly linked to feelings of romantic love. Conversely, no specific regions of the brain were notably active when 'ugly' pieces were repeated.

5 http://www.wellcome.ac.uk/news/media-office/press-releases/2011/wtvm052038.htm

What is mathematical beauty?

Professor Zeki and Dr Ishizu's research suggests beauty is an abstract concept in the mind - one that exists irrespective of the source material. It didn't take long for Professor Zeki to link this discovery to the beauty of mathematics. Three years after publishing his previous research, Professor Zeki performed a similar experiment to test how mathematicians ranked the intellectual beauty of famous formulae[6]. Fifteen mathematicians were given 60 formulae and asked to rate them 'beautiful', 'indifferent' and 'ugly' before undergoing an fMRI scan.

Again, the medial orbitofrontal cortex lit up among participants who were shown the formulae they had previously described as beautiful. But which ones were deemed the most beautiful and why? According to the results, Leonhard Euler's identity, the Cauchy-Riemann equations and the Pythagorean identity consistently topped the rankings. Of these, Leonhard Euler's identity was considered the most beautiful, while Srinivasa Ramanujan's infinite series was the 'ugliest'.

"To many of us, mathematical formulae appear dry and inaccessible, but to a mathematician an equation can embody the quintessence of beauty."

"The beauty of a formula may result from simplicity, symmetry, elegance or the expression of an immutable truth. For Plato, the abstract quality of mathematics expressed the ultimate pinnacle of beauty."

Professor Zeki

6 https://www.ucl.ac.uk/news/news-articles/0214/13022014-Mathematical-beauty-activates-same-brain-region-as-great-art-Zeki

Finding the beauty in mathematics

While the Wellcome Trust study falls short of identifying what exactly makes a formula beautiful, the Euler identity was favourably compared to Hamlet's soliloquy. It is also not the first time this particular formula has ranked highly for its aesthetic value; a 1988 issue of scholarly journal 'Mathematical Intelligencer' revealed that its readers had voted Euler's identity as mathematics' most beautiful theorem[7]. For many, Euler's identity is admired because it uses three complex numbers (e, pi and i), as well as three basic mathematical operations (addition, multiplication and exposition). The formula also utilises five core constants: zero, one, pi, i and e.

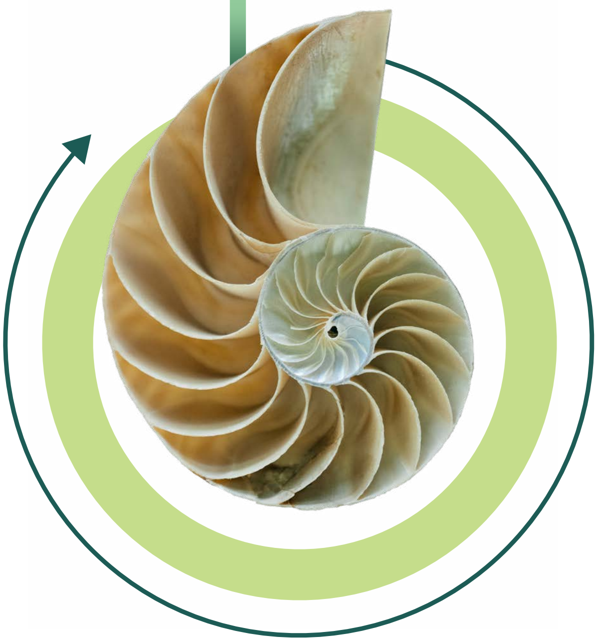
The ability to link seemingly disparate mathematical areas and create complex relationships between them, while maintaining simplicity and succinctness of structure, is a common factor in beautiful formulas. These characteristics also exist in other elegant equations, such as Einstein's Theory of General Relativity and Pierre de Fermat's Two Square theorem. Our fascination with the beauty of mathematics isn't just an intellectual pursuit, however. Mankind frequently discovers mathematical concepts in nature and has drawn inspiration from numbers and formulas for artistic purposes for thousands of years. The search for mathematical beauty is as popular now as it ever was.

7 Page 2. Dr. Euler's Fabulous Formula: Cures Many Mathematical Ills. https://books.google.co.uk/books?id=GvSg5HQ7WPcC&pg=PA2&redir_esc=y#v=onepage&q&f=false

Mathematics and nature

The natural world has always been a source of wonder for humans. We regularly reproduce its beauty in writing, music and art, and often seek respite from our hectic technology-driven lives to get closer to the great outdoors. However, just below the surface, mathematics plays a key role in shaping many of the awe-inspiring sights that we take for granted in our natural surroundings. For example, fractal geometry accurately reflects the chaos and irregularities so common in nature, such as the uniqueness of individual snowflakes or the growth patterns of trees.

Fibonacci sequences also crop up in nature with unusual regularity. Count the seed spirals in a sunflower and you'll find the total is typically a Fibonacci number. Similarly, the number of petals on many flowers is a Fibonacci and the arrangement of leaves on plants often follows Fibonacci sequences. Lilies and irises have three petals, buttercups have five, delphiniums have eight, marigolds have 13 and so on. Other examples of Fibonacci sequences in the natural world include the spiral of Nautilus shells, the structures of pinecones and Romanesco broccoli, and the reproductive dynamics of honeybees[8-9]. Romanesco broccoli is an interesting example of a fractal, as its florets are near perfect representations of the vegetable as a whole[10].



8 http://www.goldennumber.net/golden-ratio-myth/
9 Pages 523-528. Mathematics: A Practical Odyssey. David Johnson, Thomas Mowry
10 Page 15. A Simple Explanation of Absolutely Everything. Cyd Ropp.

The Golden Ratio: The epitome of beauty?

Fibonacci numbers are also linked to arguably the most well-known example of mathematical beauty - the Golden Ratio. Represented by the Greek letter Phi (Φ), the Golden Ratio was first described by mathematician Euclid in his treatise 'Elements' and has become famous for its supposed aesthetic value. The ratios of successive Fibonacci number pairs eventually converge on Phi (1.618 ...), with the ratio frequently used in art, music and architecture.

Golden rectangles - rectangles that have side lengths corresponding with the Golden Ratio - have been used to map physical beauty in human faces and bodily proportions for centuries.

Traditionally, experts believed people who are perceived as naturally beautiful have facial features that are more consistent with Phi than those whose features do not fit the ratio. This theory remains an area of contention between mathematicians and scientists, with some still confident of its validity and others claiming it has been debunked. University of Toronto researchers have even proposed a new Golden Ratio for female beauty[11].

Whether or not Phi truly corresponds with aesthetic beauty is uncertain, but the ratio has still influenced thousands of years' worth of artistic creations. The fact that mathematicians, historians and other researchers continue to debate the existence of the Golden Ratio in both natural and man-made phenomena lends credibility to its status as one of mathematics' most beautiful numbers.

11 http://www.news.utoronto.ca/researchers-discover-new-golden-ratios-female-facial-beauty-0

WILEY

Famous examples of the Golden Ratio

Advocates of the Golden Ratio's beauty have argued it was used in the creation of numerous historical masterpieces over time, ranging from the Great Pyramid of Giza and the Parthenon to the music of Mozart and Leonardo da Vinci's Vitruvian Man. Some of these claims have more substance than others; nevertheless, there is strong evidence that the ratio is present in many popular pieces of art, music and architecture.

THE CREATION OF ADAM

Painted on the ceiling of the Sistine Chapel, this is perhaps Michelangelo's most famous work. God and Adam's fingertips appear to meet at the Golden Ratio point of the width and height of the golden rectangles that contain both figures[12].

THE SCHILLINGER SYSTEM

Joseph Schillinger was among the first music theorists to formulate a system that used the Golden Ratio and Fibonacci numbers to determine the parameters of musical composition[13]. His contributions laid the groundwork for the application of mathematics and physics to musicology.

THE UNITED NATIONS (UN) SECRETARIAT BUILDING

Completed in 1952, the centrepiece of the UN's headquarter complex in New York is widely thought to follow the Golden Ratio. One of the designers, French architect Le Corbusier, invented the Modulor anthropometric scale of proportions, which was based on Fibonacci numbers and Phi[14]. Golden rectangles are identifiable across both interior and exterior aspects of the building's design, including the entrance and floor layouts.

12 http://onlinelibrary.wiley.com/enhanced/doi/10.1002/ca.22580/
13 https://helda.helsinki.fi/bitstream/handle/10138/26236/onmusica.pdf
14 http://journal.eahn.org/articles/10.5334/ah.by/

Why mathematical beauty matters

Euler's identity, Einstein's Theory of Relativity, Fibonacci sequences, the Golden Ratio - why is it important to acknowledge and promote the aesthetic value of these iconic numbers, formulas and theorems?

After all, practitioners no doubt already appreciate their mathematical elegance, so isn't this simply preaching to the choir? For many mathematicians, particularly those tasked with encouraging new generations of students to pursue the subject, the answers to these questions are usually more practical than abstract. Namely, the beauty of mathematics can make a challenging discipline more accessible and engaging for young people.

Whether it's unique number patterns, the existence of key formulae in nature or the intriguing links between mathematics and famous masterpieces, educators should utilise every tool at their disposal to optimise student engagement. Annual international conferences such as the Bridges: Mathematical Connections in Art, Music and Science event highlight the interdisciplinary opportunities that exist between the various fields in an effort to build interest in mathematics as a creative, aesthetic pursuit. This approach can breathe new life into mathematics in the classroom by evoking curiosity in students and showing how important the subject is when it comes to understanding the world - and universe - around us.

Conclusion

There is a beauty in numbers, equations and shapes that mathematicians instinctively understand. Only recently, however, have researchers shown that beautiful mathematics evokes the same pleasure response in the brain as breathtaking pieces of art or stunning musical compositions. While we are perhaps only now beginning to accurately quantify the discipline's aesthetic value, the presence of key mathematical concepts has been noted in nature and replicated in art throughout history. Recognising and celebrating this beauty is not only an eye-opening experience but also a potential method of unlocking some of the wonders of mathematics for the next generation of mathematicians.

Famous quotes

"Mathematics, rightly viewed, possesses not only truth, but supreme beauty - a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. The true spirit of delight, the exaltation, the sense of being more than Man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as poetry."

BERTRAND RUSSELL (1872–1970)

British mathematician and philosopher

"What makes the theory of relativity so acceptable to physicists in spite of its going against the principle of simplicity is its great mathematical beauty. This is a quality which cannot be defined, any more than beauty in art can be defined, but which people who study mathematics usually have no difficulty in appreciating."

PAUL DIRAC (1902–1984)

British theoretical physicist

"The aesthetic goals and the utilitarian goals for mathematics turn out, in the end, to be quite close. Our aesthetic instincts draw us to mathematics of a certain depth and connectivity. The very depth and beauty of the patterns makes them likely to be manifested, in unexpected ways, in other parts of mathematics, science, and the world."

WILLIAM THURSTON (1946–2012)

American mathematician and Fields Medal winner

"My work always tried to unite the truth with the beautiful, but when I had to choose one or the other, I usually chose the beautiful."

HERMANN WEYL (1885–1955)

German mathematician and theoretical physicist

WILEY

The beauty of numbers

A celebration of how mathematics shapes our world

wiley.com